Sectioned Convolution

and

SCDWT

分段摺積 與 分段摺積離散小波

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i. Abstract

ii. Sectioned Convolution

iii. Sectioned Convolution in DWT

iv. Efficiency Comparison

v. Future Work

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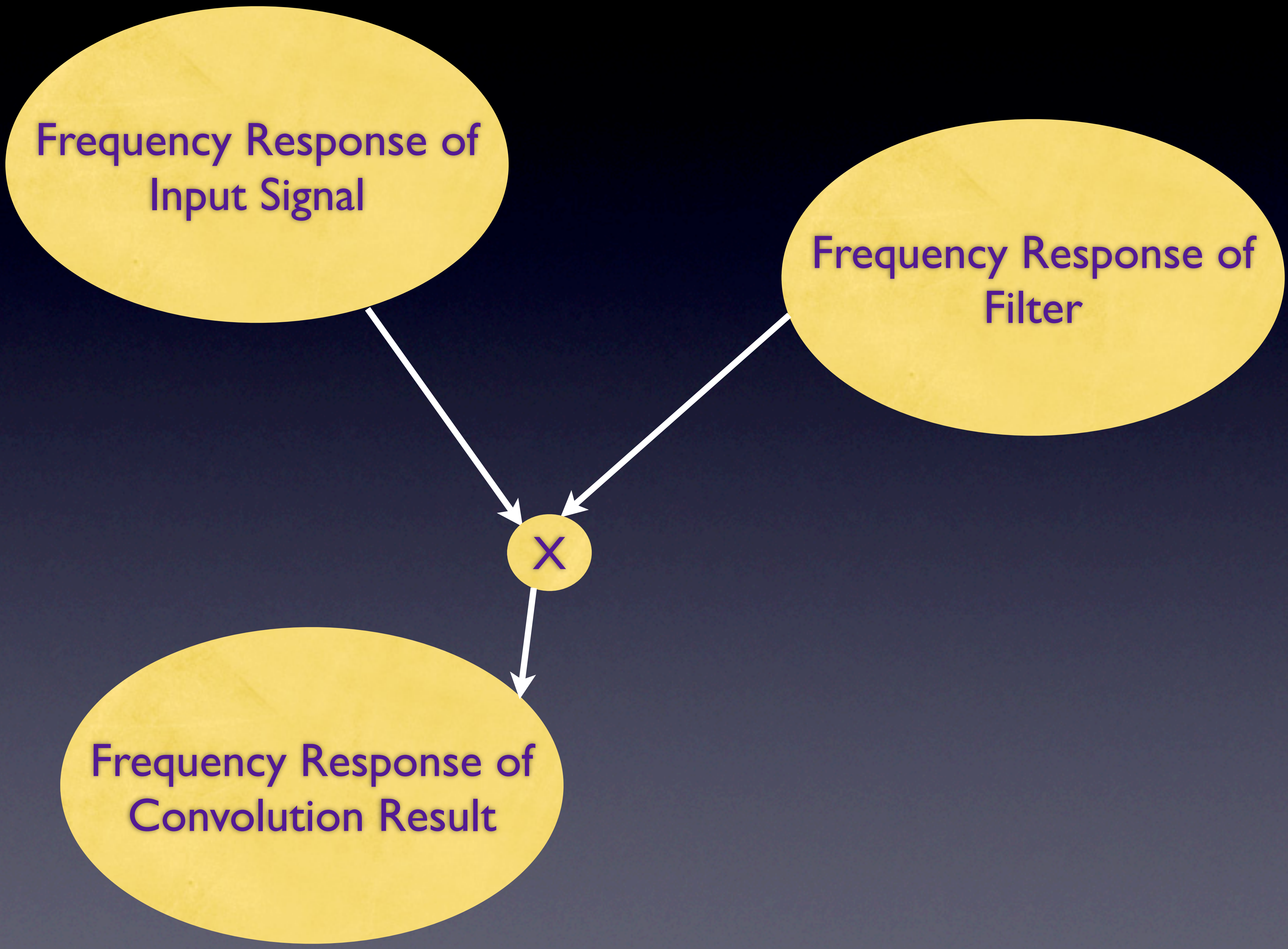
Sectioned Convolution

how to perform the
“ convolution ” !?


Frequency Response of Input Signal
Frequency Response of Filter
X
Frequency Response of Convolution Result

Here is a problem.....

How many points of FFT should be calculated !?

Signal Length

$$N + M - 1$$

Filter Length

How to calculate the FFT !?

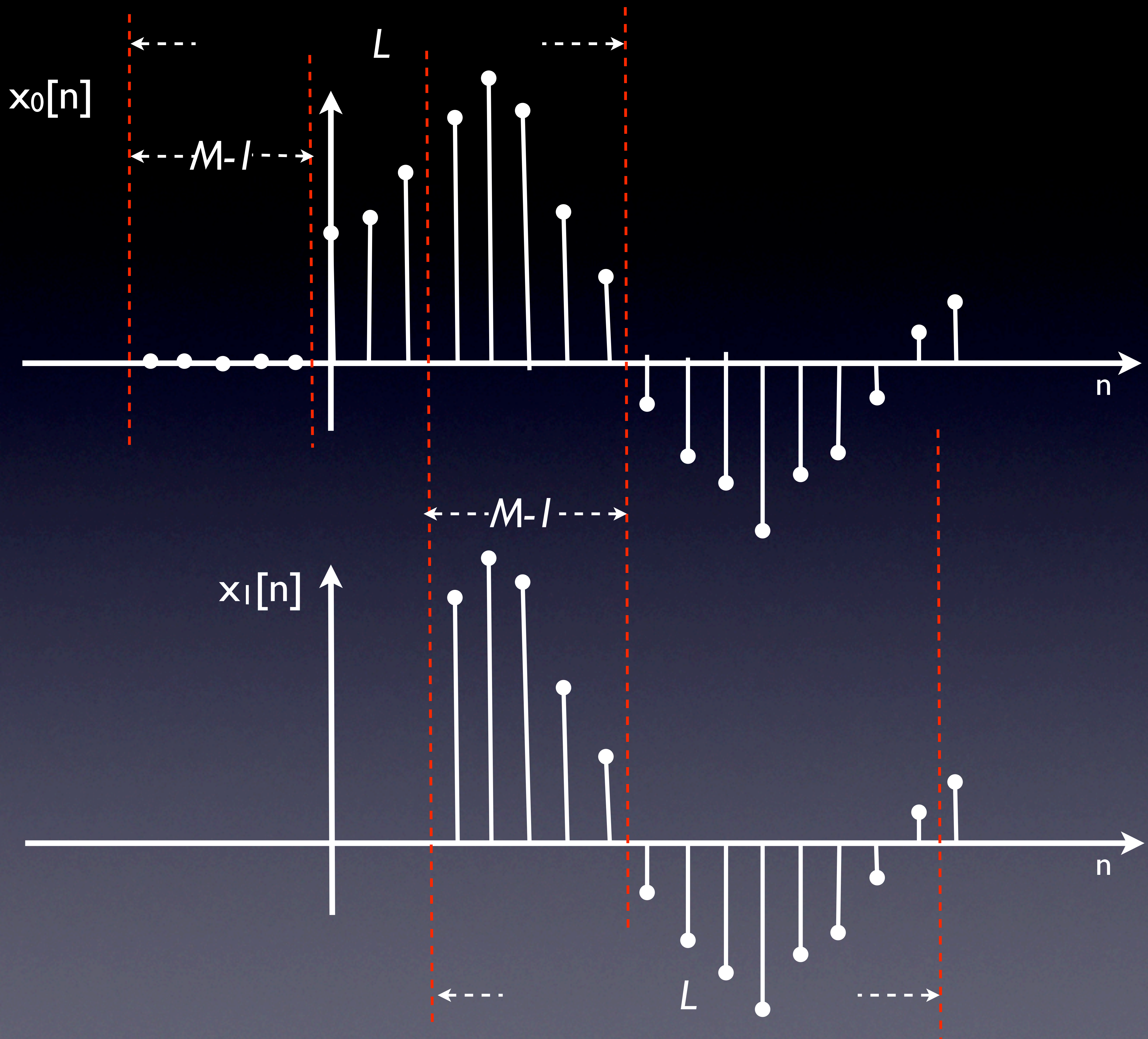
	Mults	Adds
Radix - 2	$N/2 \log_2 N$	$N \log_2 N$
Split - Radix	$N/4 (\log_2 N+1)$	$3N/4 (\log_2 N+1/3)$

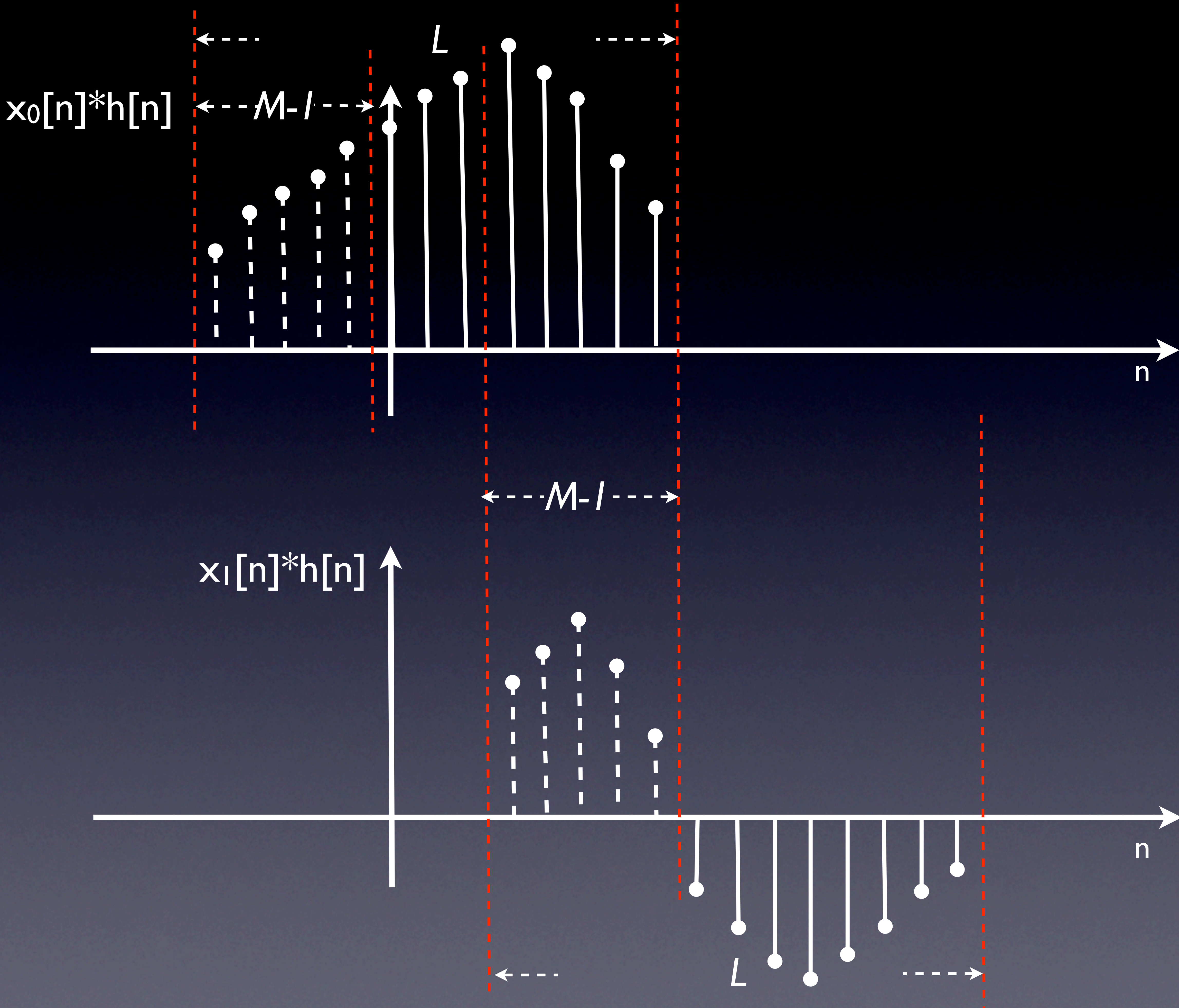
what is the mean of

" sectioned " !?

we split the input signal into
section by section.

Overlap-Saved Method


L
x0[n]
M-1
n
M-1
x1[n]
n
L



L
x0[n]*h[n]
M-1
n
M-1
x1[n]*h[n]
n
L

what is the advantage of
overlap-saved method !?

i. we do not have to wait until all of the signal has been received.

ii. it does not increase the system complexity.

Sectioned Convolution

The complexity of sectioned convolution is

$$C = 2 * \frac{N}{L - M + 1} \frac{L}{2} \log_2 L$$

L : sectioned length, M : filter length

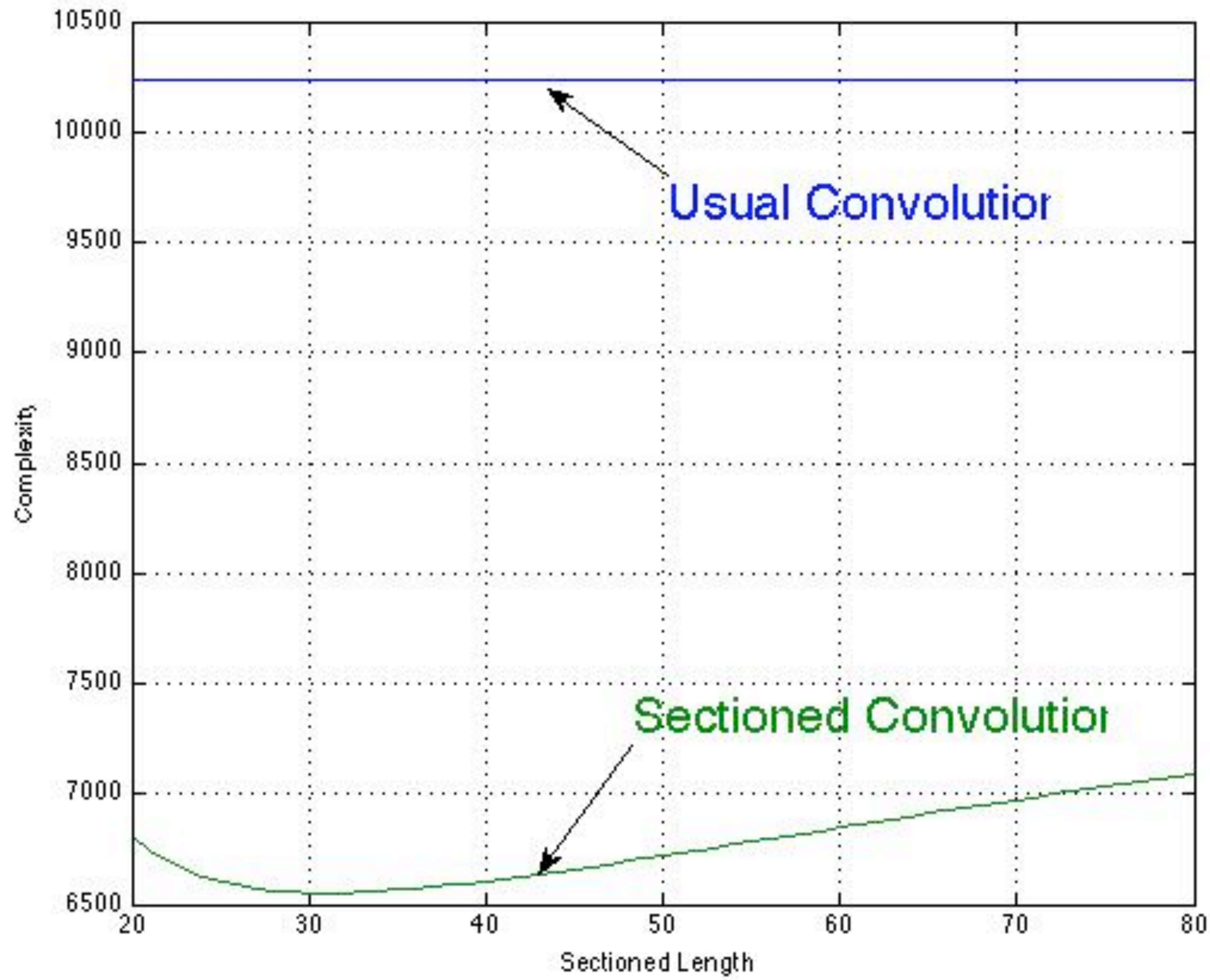
The optimal sectioned length of sectioned convolution is

$$M = \frac{L + 1 + \log L}{1 + \log L} = 1 + \frac{L}{1 + \log_2 L}$$

L : sectioned length, M : filter length

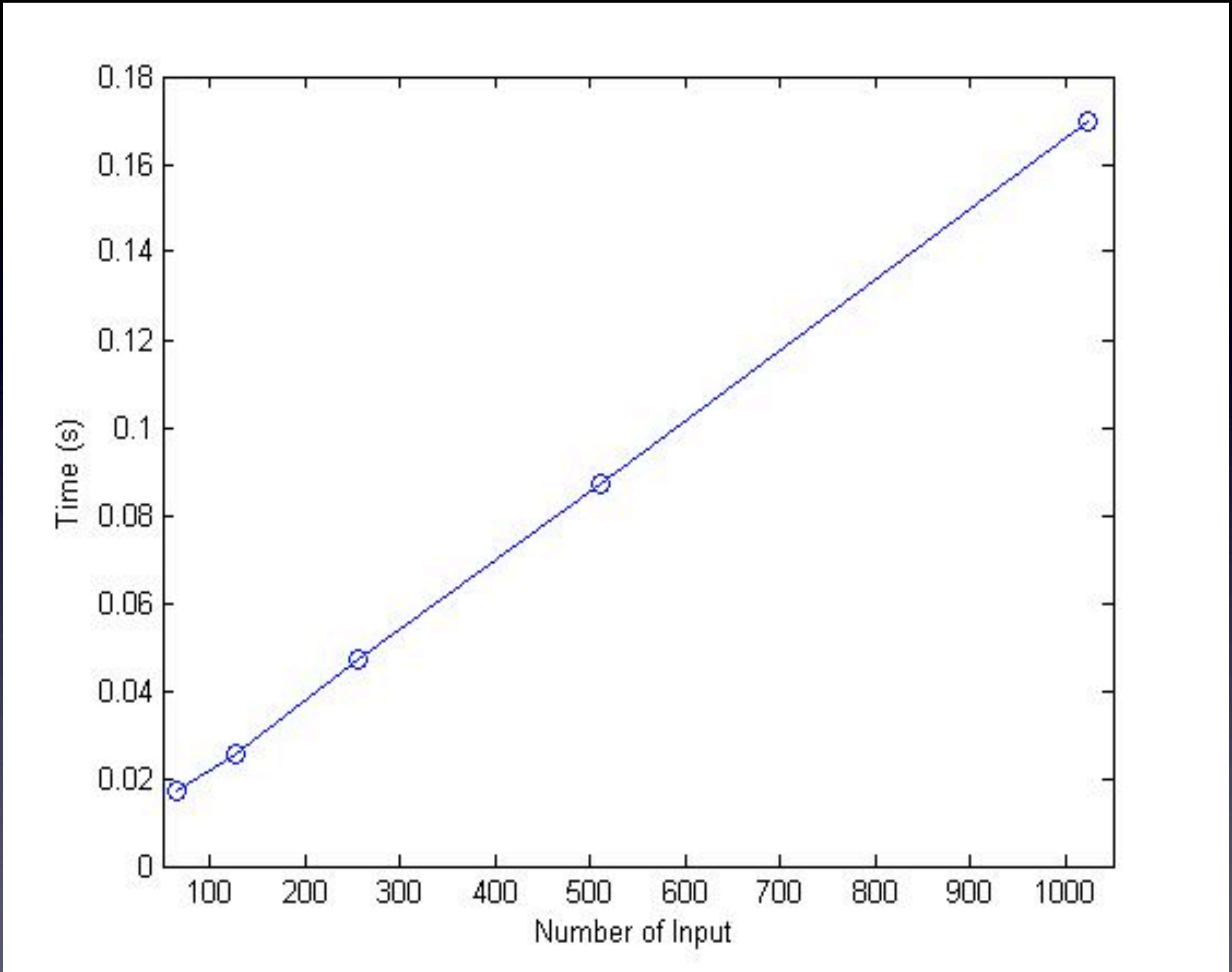
what is the advantage of
sectioned convolution !?

i. Saving Energy

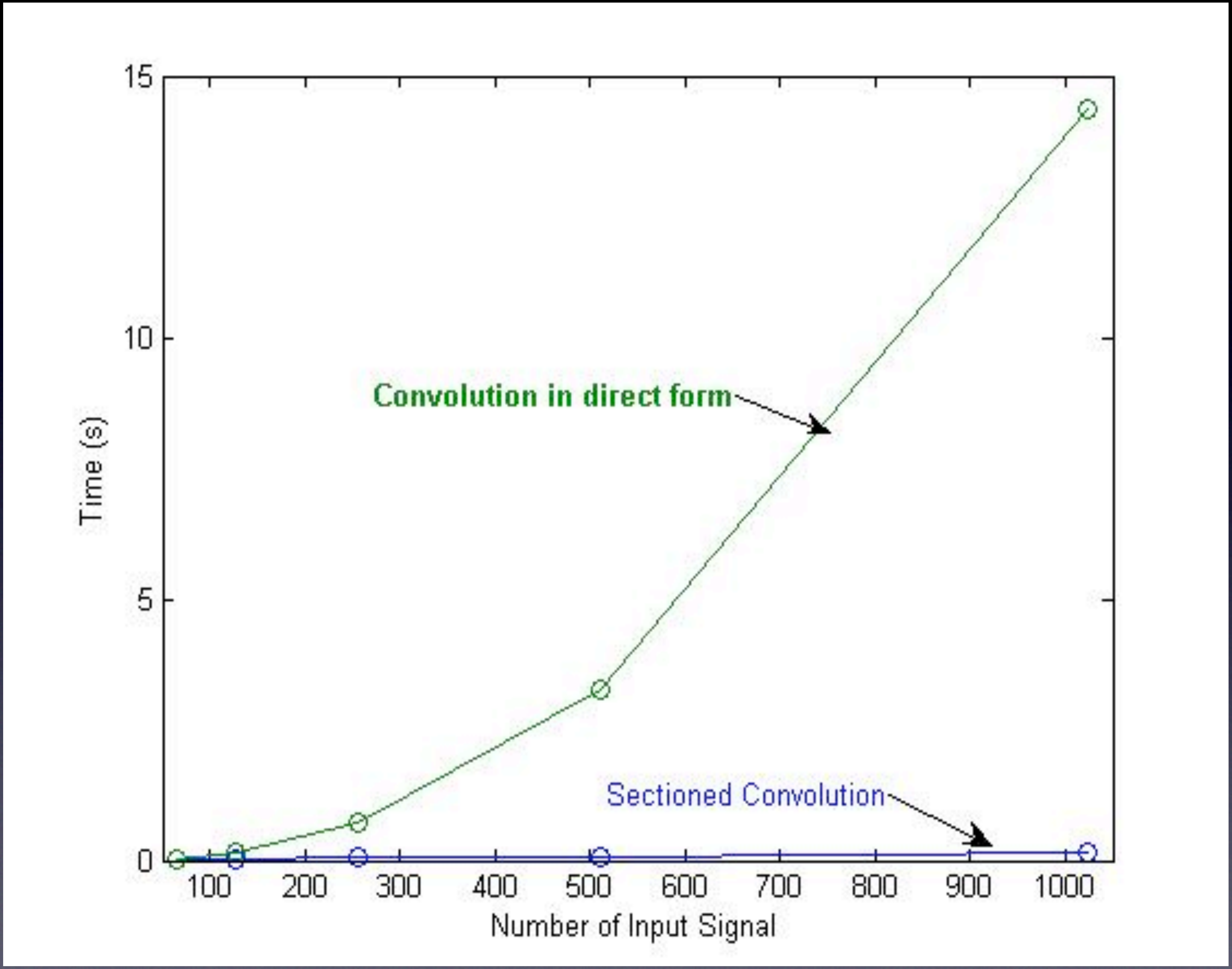


10500
10000
9500
9000
8500
8000
7500
7000
6500
Complexity
20
30
40
50
60
70
80
Sectioned Length
Usual Convolutior
Sectioned Convolutior

ii. Saving Time



0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Time (s)
100
200
300
400
500
600
700
800
900
1000
Number of Input



Convolution in direct form
Sectioned Convolution
Time (s)
Number of Input Signal
15
10
5
0
100
200
300
400
500
600
700
800
900
1000

iii. Fixed Hardware Architecture

The Complexity of Sectioned Convolution is

$$C = 2 * \frac{N}{L - M + 1} \frac{L}{2} \log_2 L$$

Optimal Sectioned Length is

$$M = \frac{L + 1 + \log L}{1 + \log L} = 1 + \frac{L}{1 + \log_2 L}$$

No matter how long the input signal is,
the points of FFT depends on the filter length.

i. Saving Energy

ii. Saving Time

iii. Better Hardware Architecture

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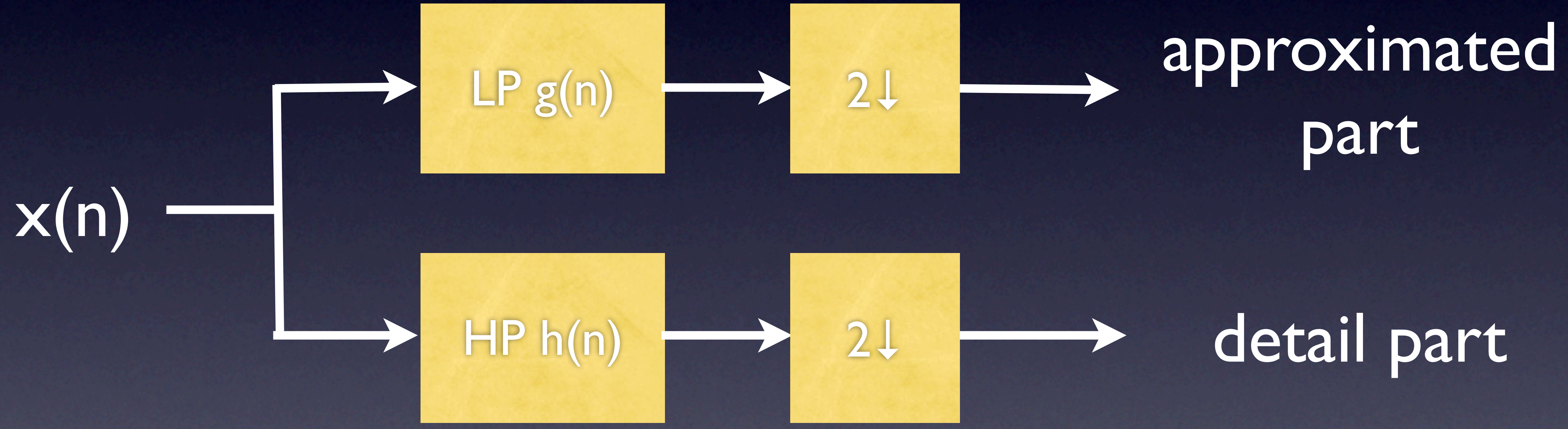
SCDWT

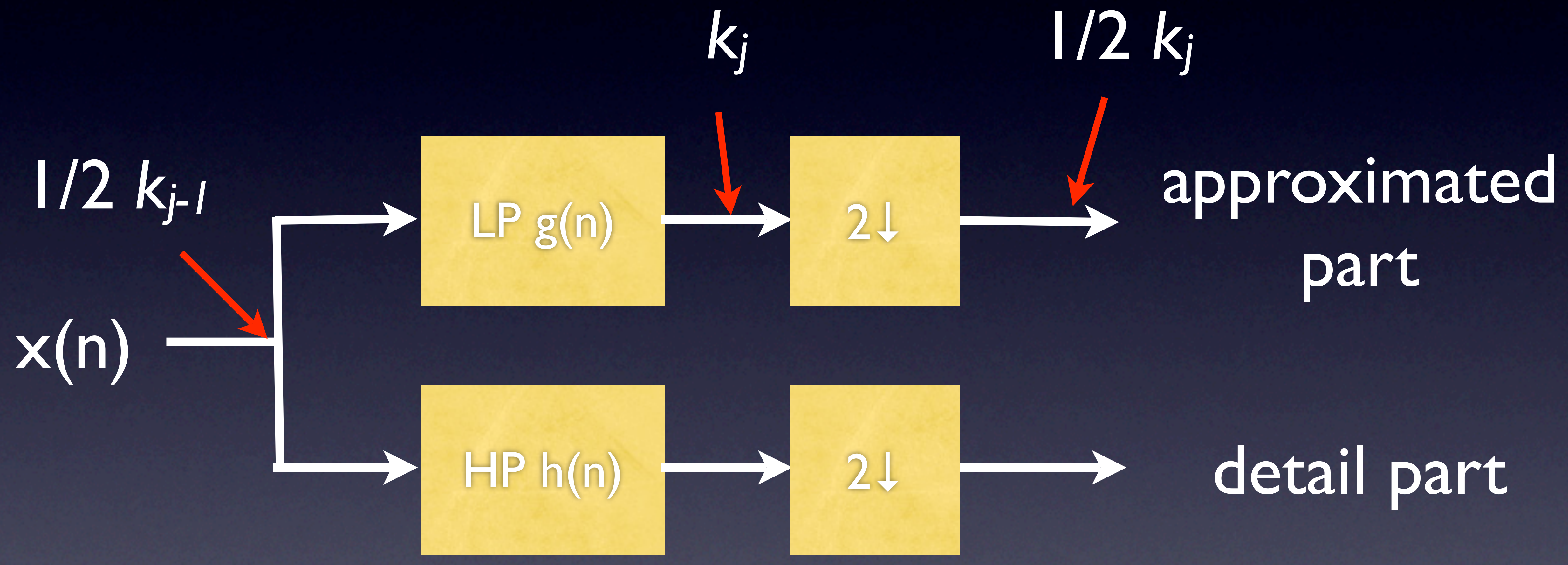
1-Dimension DWT



$$k_j = k_{j-1}/2 + M - 1 \qquad j = 1, 2, 3, \ldots$$

* k_j is the input length in each level

1-Dimension DWT



The Complexity of 1-Dimension DWT

1time k_j-point FFT for input signal
+ 2 times k_j-point FFT for filters
+ 2 times k_j-point FFT for DWT outputs

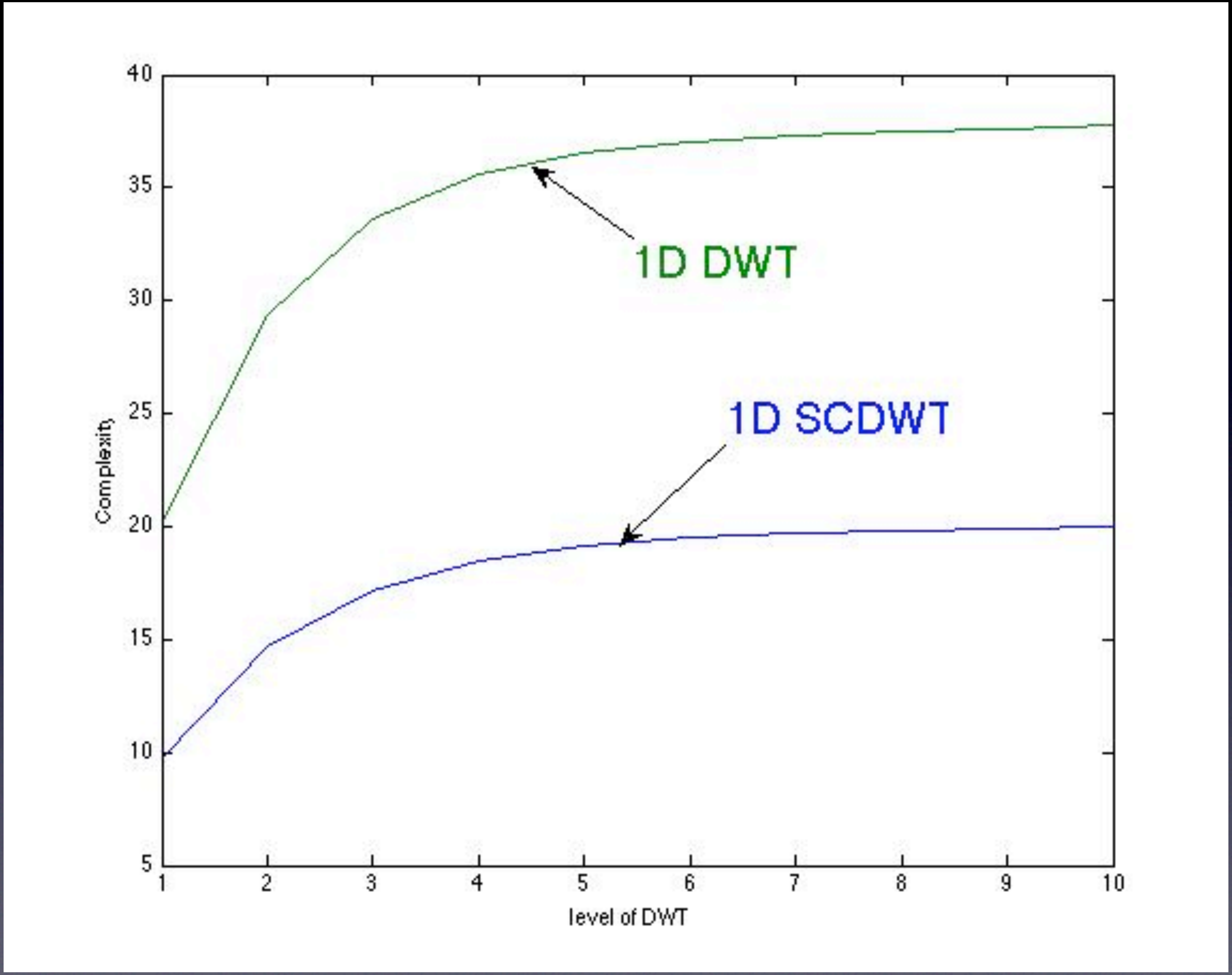
$$C_{uc} = 5 * \sum_{j=1}^{L} \frac{1}{2} k_j \log_2 k_j = 2.5 * \sum_{j=1}^{J} k_j \log_2 k_j$$

The Complexity of 1-Dimension SCDWT

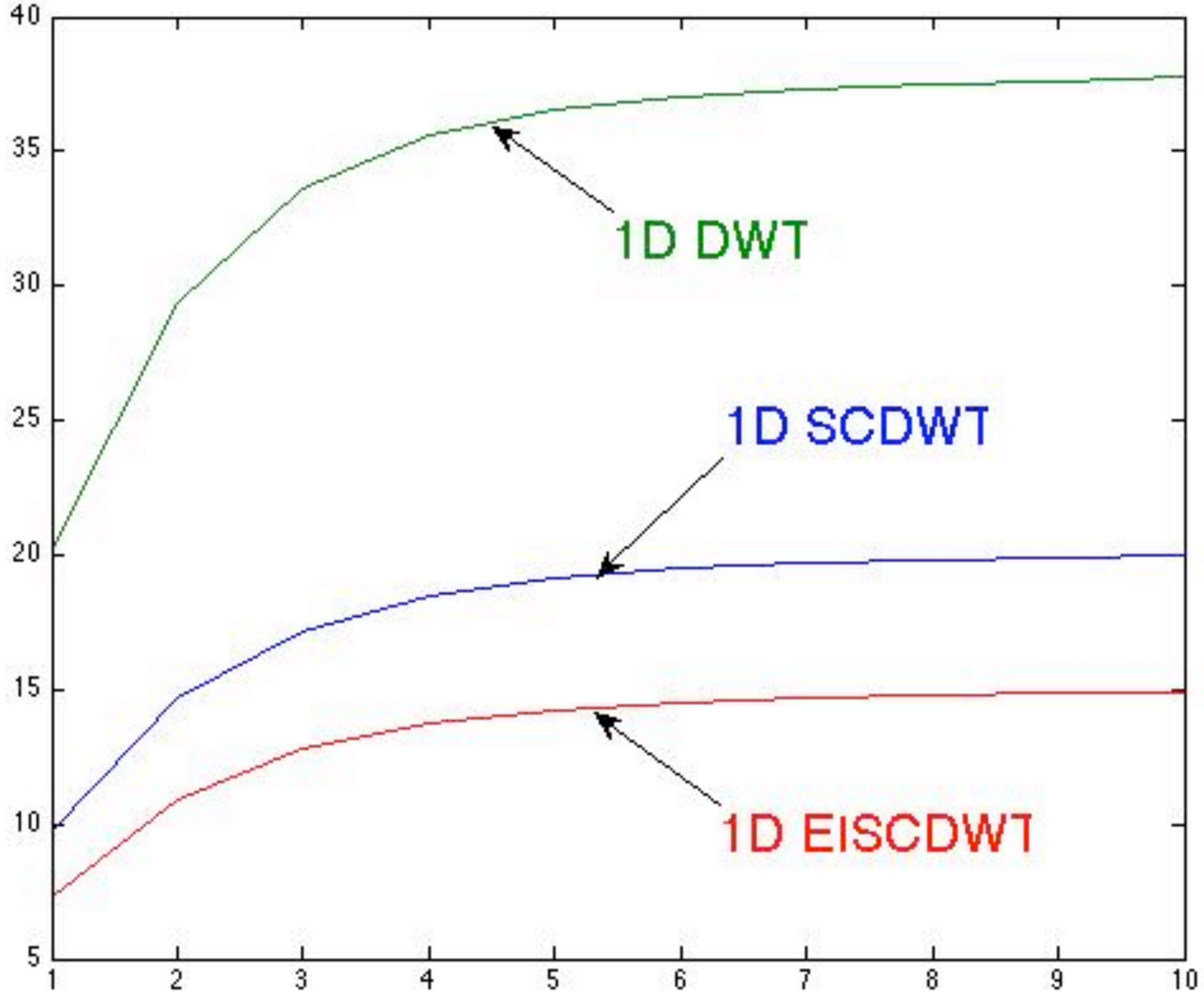
1/2 k_{j-1} times L-point FFT for input signal
+ 2 times L-point FFT for filters
+2*1/2 k_{j-1} times L-point FFT for DWT outputs

$$C_{SC} = L\log_2 L + 3*\sum_{j=1}^{J}\left(\frac{\frac{1}{2}k_{j-1}}{L-M+1}\frac{L}{2}\log_2 L\right)$$

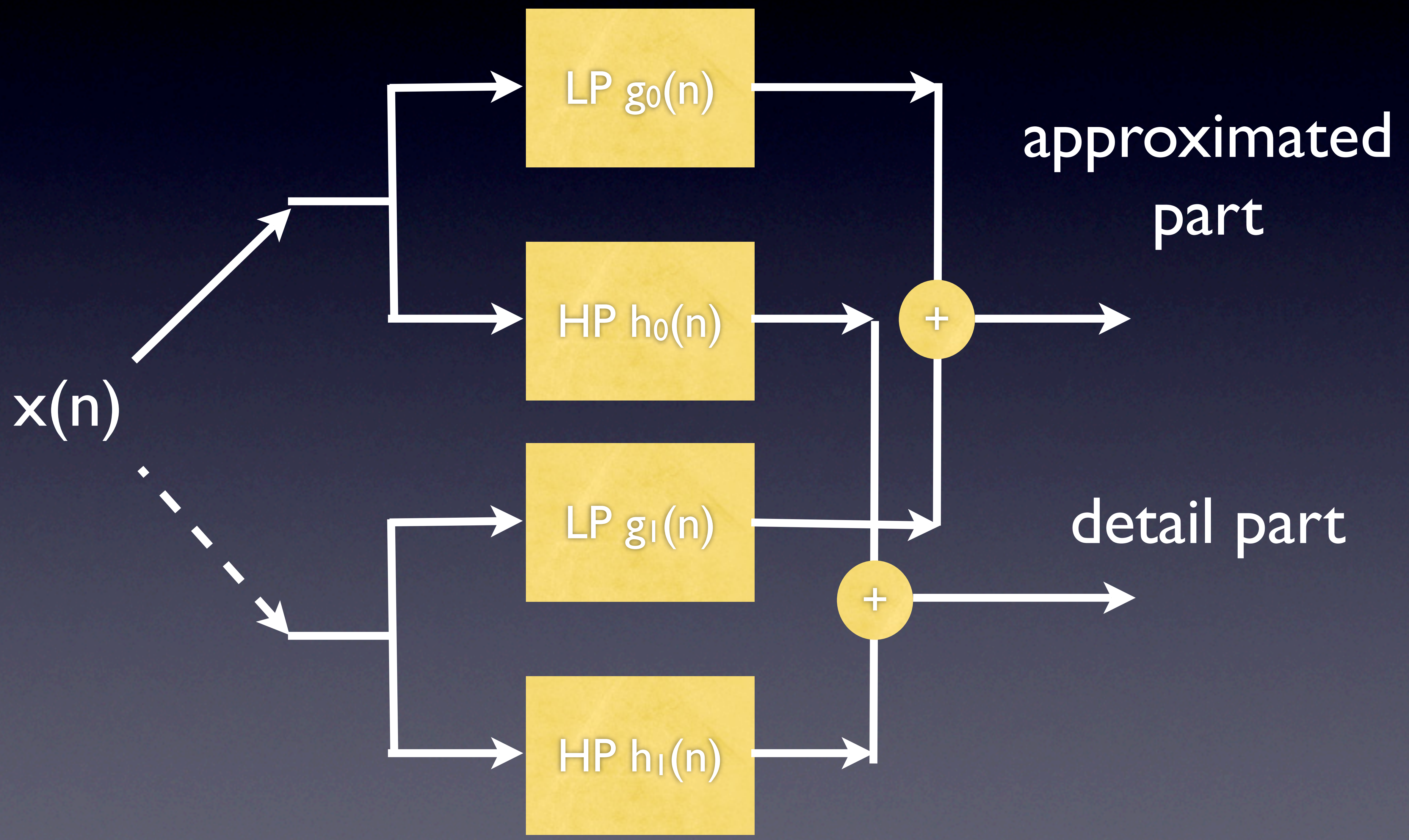
$$= L\log_2 L + \frac{3}{2}*\left(\frac{L}{L-M+1}\log_2 L\right)*\sum_{j=1}^{J}\frac{1}{2}k_{j-1}$$



1D DWT
1D SCDWT
Complexity
level of DWT
40
35
30
25
20
15
10
5
1
2
3
4
5
6
7
8
9
10


1D DWT
1D SCDWT
1D EISCDWT
40
35
30
25
20
15
10
5
1
2
3
4
5
6
7
8
9
10

1-Dimension EIDWT



The Complexity of 1-Dimension EISCDWT

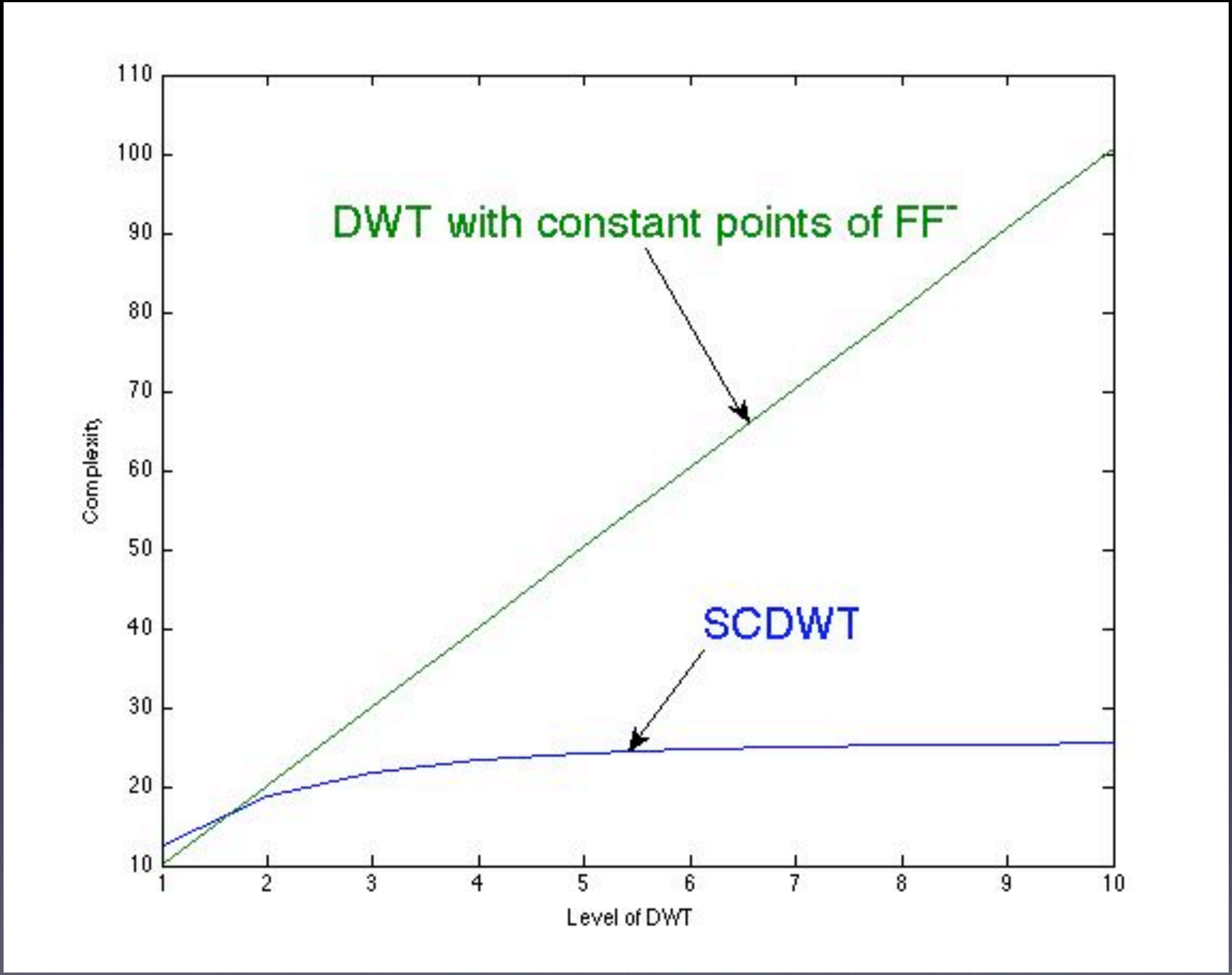
2*1/4 k_{j-1} times L`-point FFT for input signal
+ 4 times L`-point FFT for filters
+4*1/4 k_{j-1} times L`-point FFT for DWT outputs

$$C_{EISC} = 2 * L' \log_2 L' + 6 * \sum_{j=1}^{J} \left(\frac{\frac{1}{4} k_{j-1}}{L' - 0.5M + 1} \frac{L'}{2} \log_2 L' \right)$$

$$= 2 * L' \log_2 L' + 3 * \frac{L' \log_2 L'}{L' - 0.5M + 1} * \sum_{j=1}^{J} \frac{1}{4} k_{j-1}$$

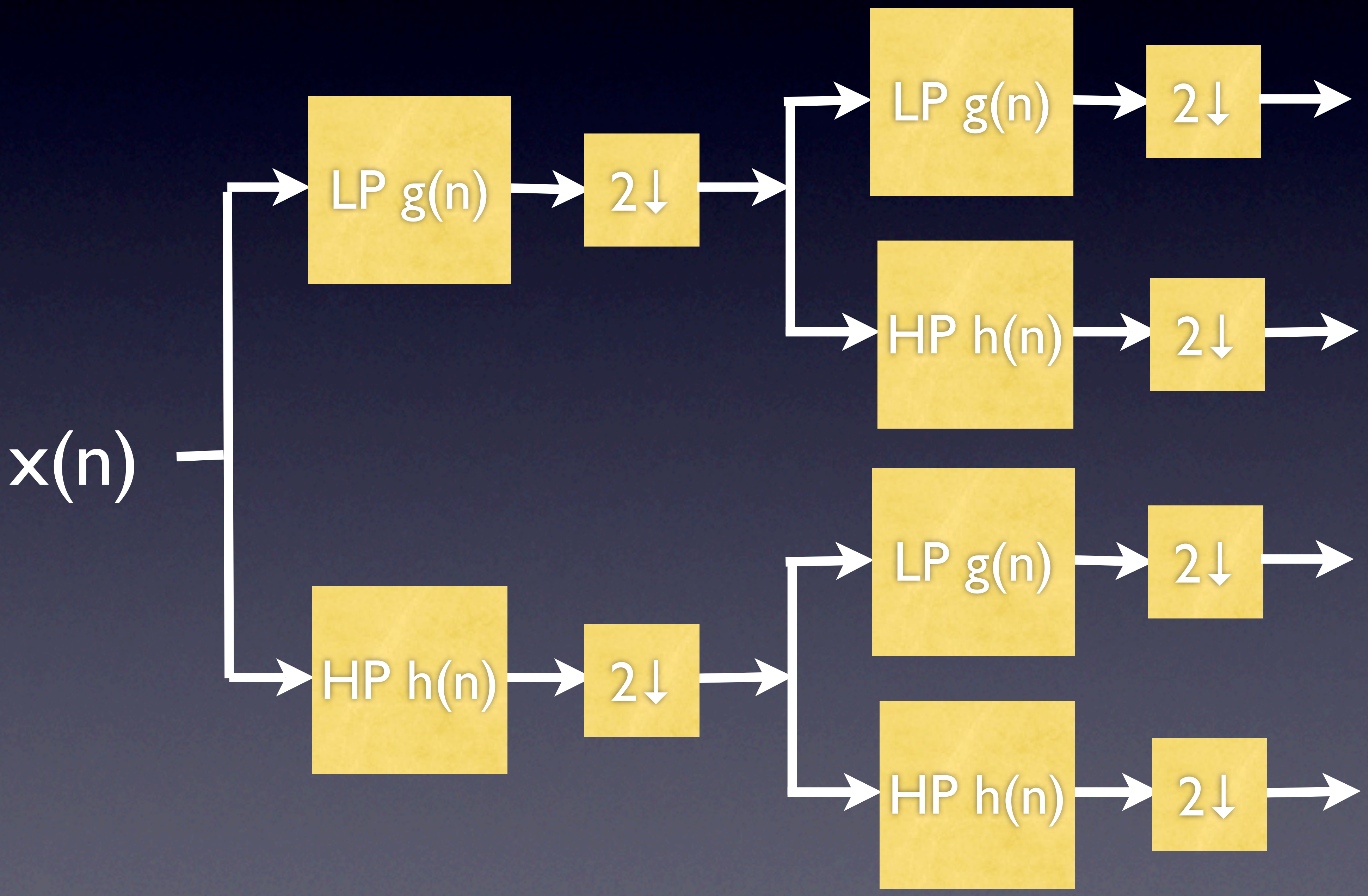
one of the advantages of SCDWT is........

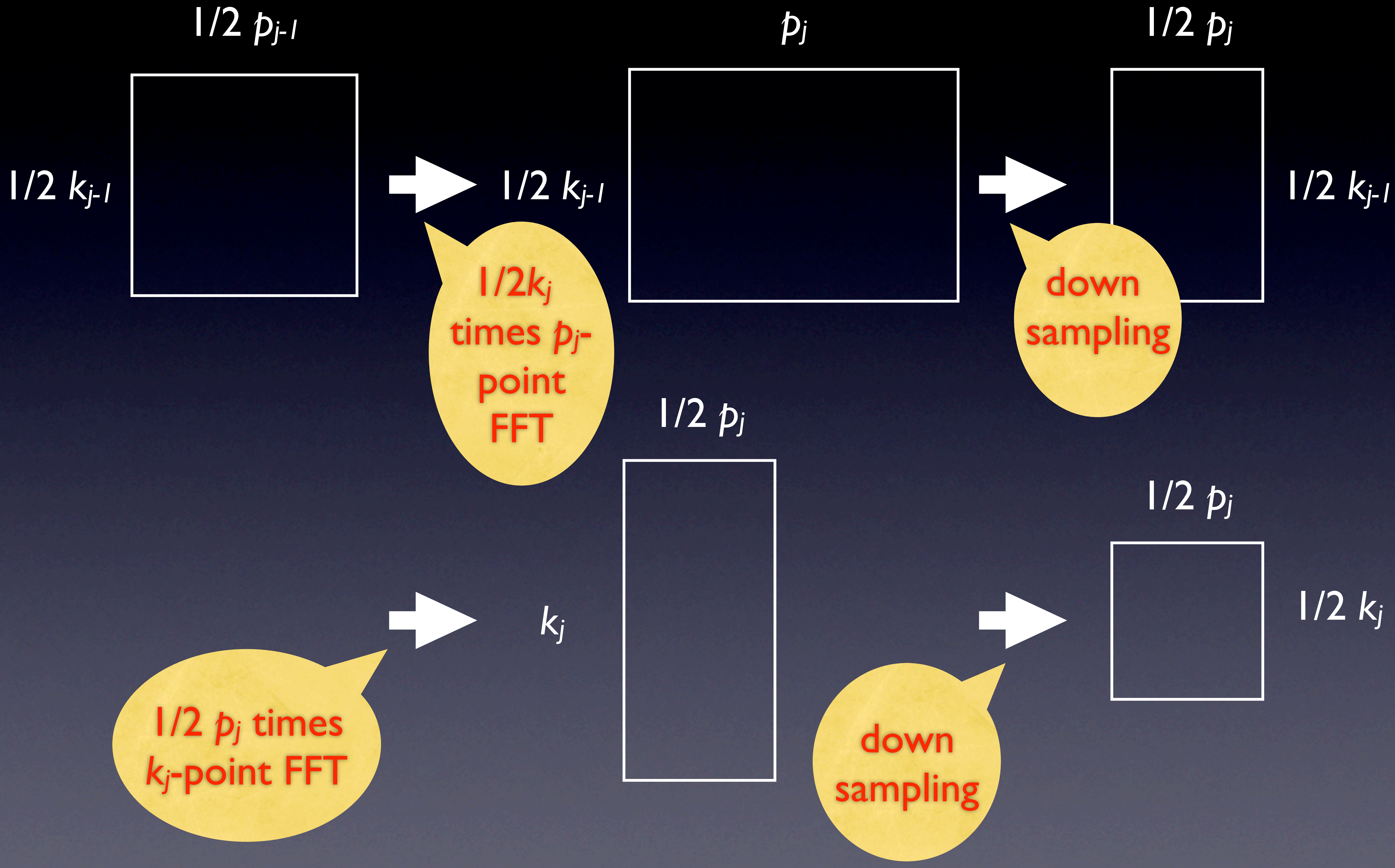
Fixed Hardware Architecture



DWT with constant points of FFT
SCDWT
Complexity
Level of DWT
110
100
90
80
70
60
50
40
30
20
10
1
2
3
4
5
6
7
8
9
10

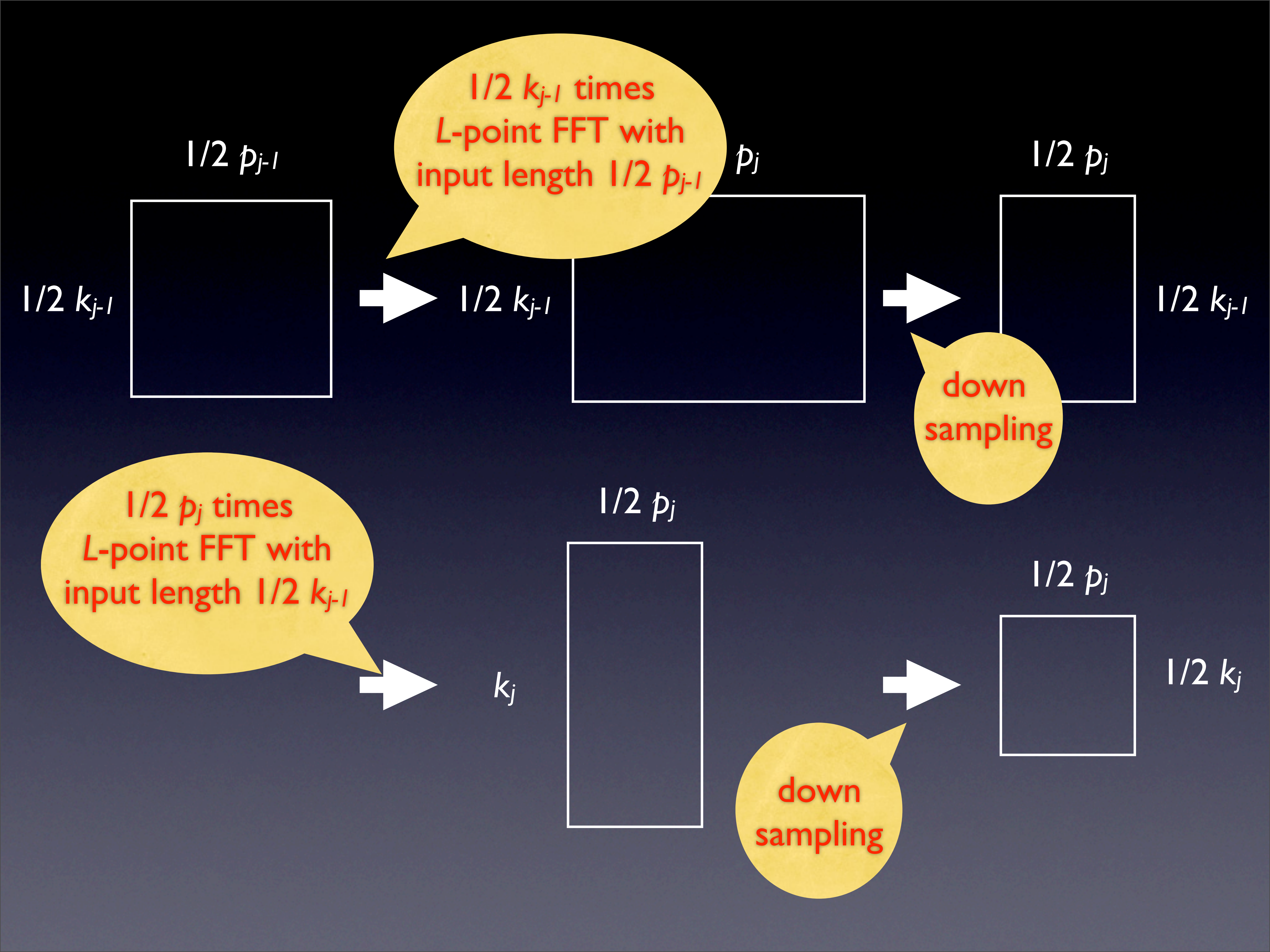
2-Dimension DWT




1/2 p_{j-1}
1/2 k_{j-1}
1/2 k_{j-1}
p_j
1/2 p_j
1/2 k_{j-1}
1/2k_j times p_j-point FFT
down sampling
1/2 p_j
k_j
1/2 p_j
1/2 k_j
1/2 p_j times k_j-point FFT
down sampling

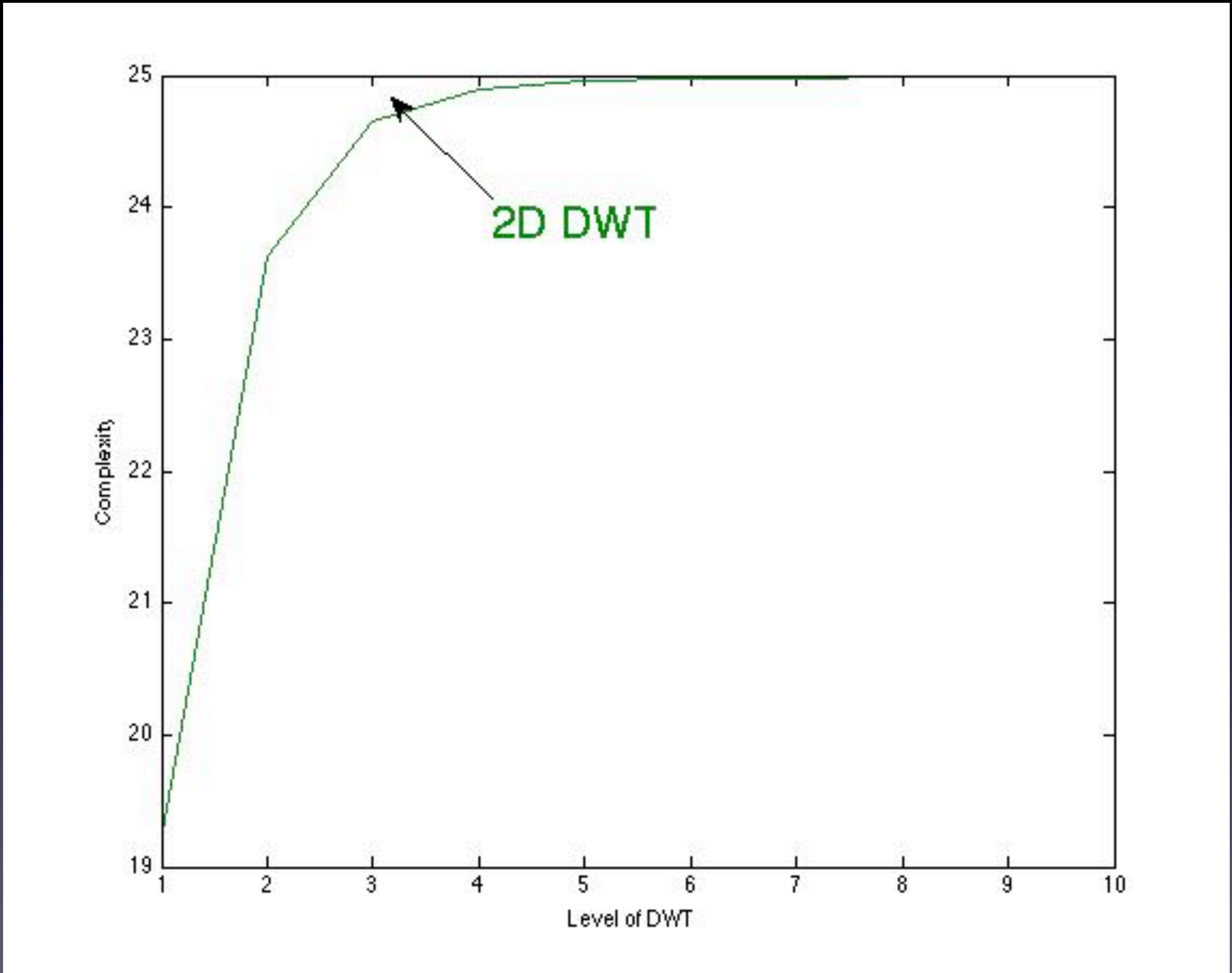
The Complexity of 2-Dimension DWT

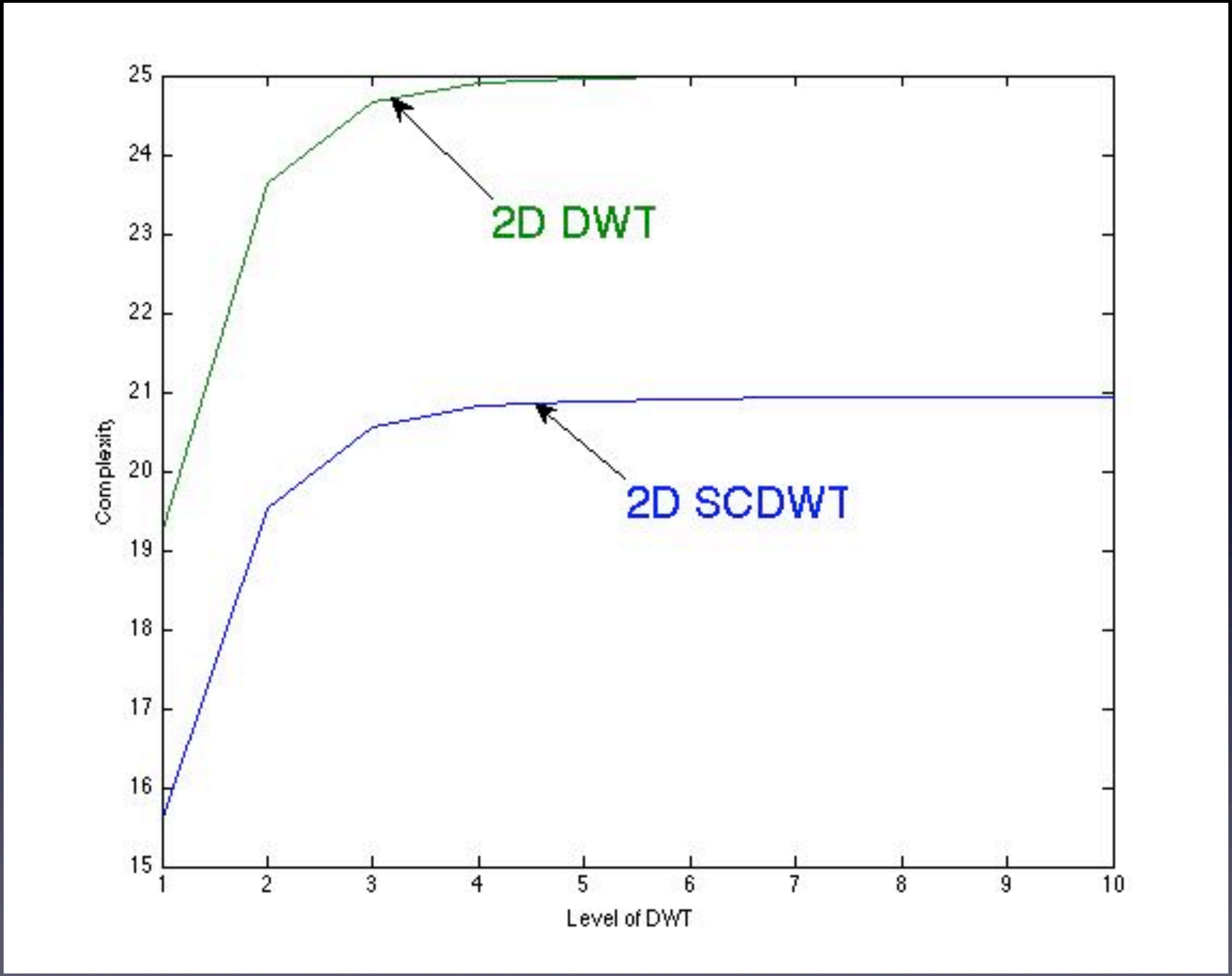
$$C_o = \frac{1}{2} * \sum_{j=1}^{L} \left(\frac{1}{2} k_{j-1} + 1 \right) p_j \log_2 p_j + 2 * \left(\frac{1}{2} p_j + 1 \right) k_j \log_2 k_j + k_j p_j \log_2 \frac{1}{4} k_j p_j$$


1/2 k_{j-1} times
L-point FFT with
input length 1/2 p_{j-1}
1/2 p_{j-1}
1/2 k_{j-1}
p_j
1/2 k_{j-1}
1/2 p_j
1/2 k_{j-1}
down
sampling
1/2 p_j times
L-point FFT with
input length 1/2 k_{j-1}
1/2 p_j
k_j
1/2 p_j
1/2 k_j
down
sampling

The Complexity of 2-Dimension SCDWT

$$C_{oSC} = L\log_2 L + \sum_{j=1}^{J} 1.5 * \frac{1}{2} k_{j-1} \frac{\frac{1}{2} p_{j-1}}{L - M + 1} L\log_2 L + 2 * \frac{1}{2} p_j \frac{\frac{1}{2} k_{j-1}}{L - M + 1} L\log_2 L$$


25
24
23
22
21
20
19
Complexity
1
2
3
4
5
6
7
8
9
10
Level of DWT
2D DWT



2D DWT
2D SCDWT
Complexity
Level of DWT
25
24
23
22
21
20
19
18
17
16
15
1
2
3
4
5
6
7
8
9
10

4

Efficiency Comparison of 1-D DWT

Input Length	DWT	SCDWT	EISCDWT
1024	36.5	19.1	14.2
512	73.0	28.4	21.2
256	146.0	47.1	35.1
128	292.1	84.5	63.0

* DWT level is fixed, Filter length is fixed

Efficiency Comparison of 1-D DWT

Input Lenght	DWT	SCDWT	EISCDWT
1024	-	52.32%	38.98%
512	-	38.96%	29.03%
256	-	32.28%	24.05%
128	-	28.94%	21.57%

* DWT level is fixed, Filter length is fixed

Efficiency Comparison of 1-D DWT

Filter Length	DWT	SCDWT	EISCDWT
8	73.0	28.4	21.2
16	75.3	35.0	28.9
32	80.1	41.9	36.1
64	90.0	50.4	44.6

* DWT level is fixed, Input length is fixed

Efficiency Comparison of 1-D DWT

Filter Length	DWT	SCDWT	EISCDWT
8	-	38.86%	29.03%
16	-	46.51%	38.36%
32	-	52.37%	45.14%
64	-	56.06%	49.56%

* DWT level is fixed, Input length is fixed

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Conclusion and Future Work

No matter how long the input signal is,
the points of FFT depends on the filter length.

Reference

[1] Gerard P. M. Egelmeers and Piet C. **W.** Sommen, “A new method for efficient convolution in frequency domain by nonuniform partitioning for adaptive filtering”, IEEE Trans. Signal Process., Vol. 44, No.12, JUNE 1996

[2] Jung Kap Kuk and Nam Ik Cho, “Block convolution with arbitrary delays using fast Fourier transform”, ISPACS 2005

[3] Guillermo García, “Optimal Filter Partition for Efficient Convolution with Short Input Output Delay”, AES 113TH CONVENTION, LOS ANGELES, CA, USA, 2002 OCTOBER 5–8

Thank you